Midterm Review

1. These slides and review points found at
   http://math.utoledo.edu/~dgajews/1180

2. Bring a photo ID card:
   Rocket Card, Driver's License

Covers:

4.1 Graphs + Euler Paths
4.2 Traveling Salesman + Hamiltonian Paths
2.1 Sets
2.2 Set Theory
2.3 Set Operations
6.1 Number Theory
11.1 Voting Methods
11.2 Defects of Voting Methods
11.3 Weighted Voting Systems

Know the basic vocabulary of the sections.

The test will be multiple choice.

The test will be like the online HW rather than the lab assignments.

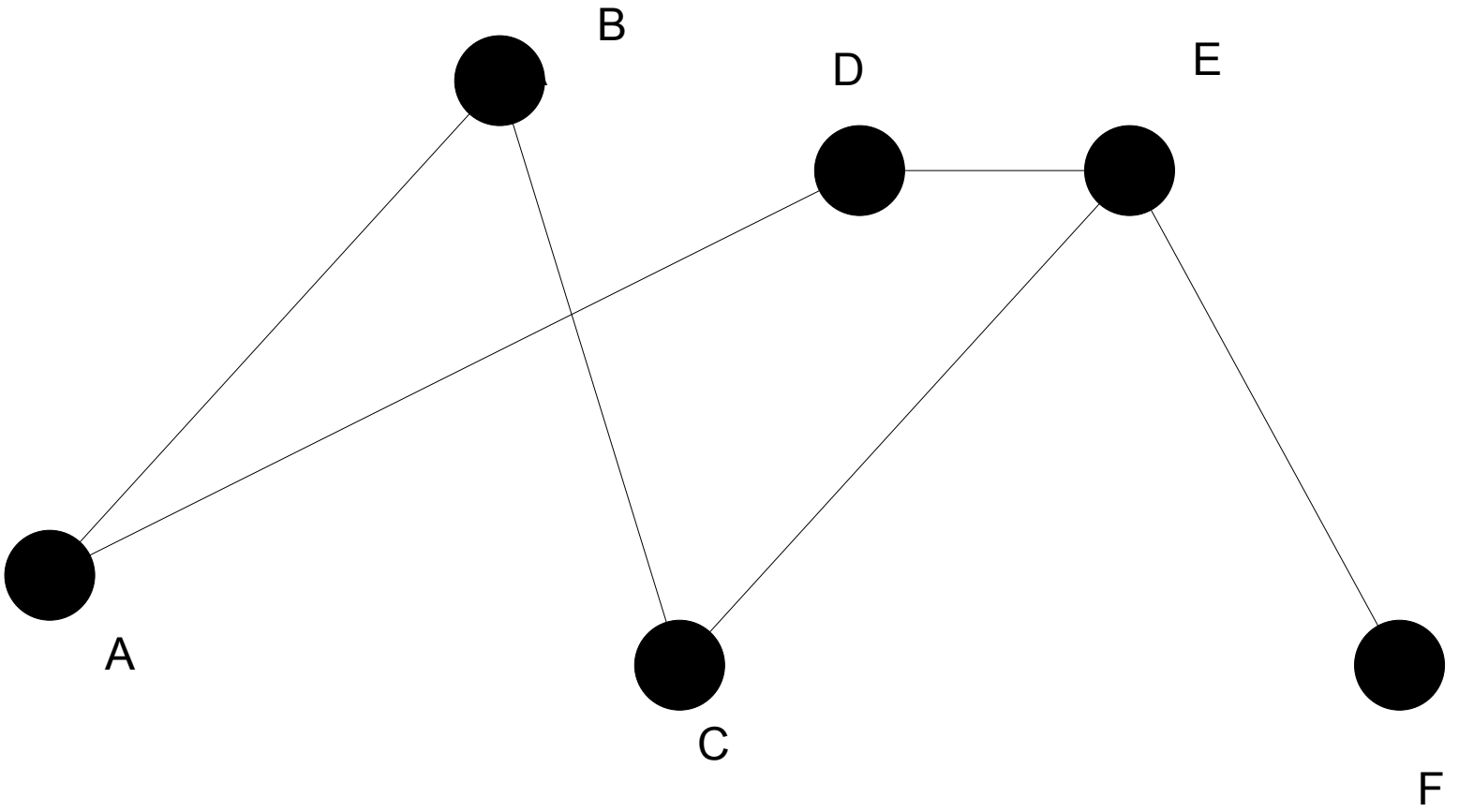
Graphs are made up of 2 parts:

v

and e

A graph is **connected** if

A **path** is a sequence of adjoined edges along a graph



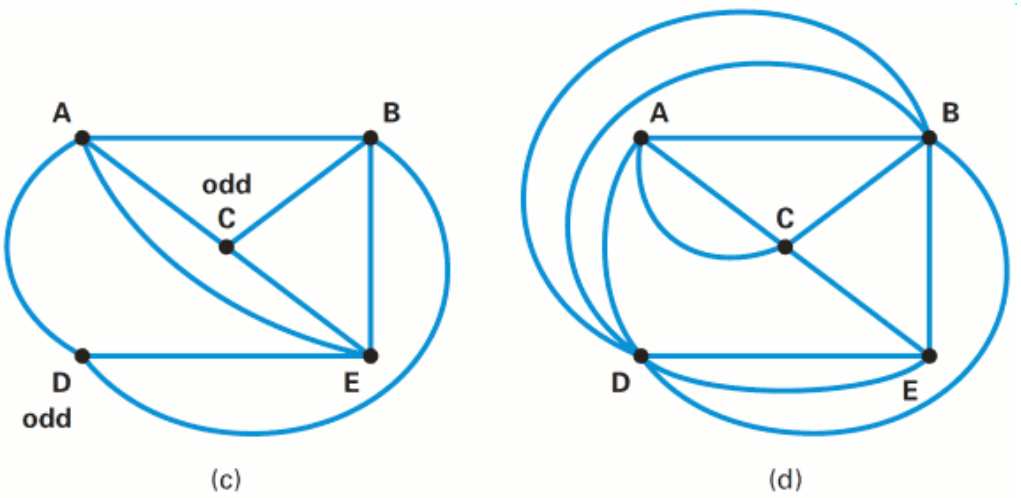
An Euler graph visits every _____ once.

Mnemonic:

Euler Graph visits every
Edge exactly once.

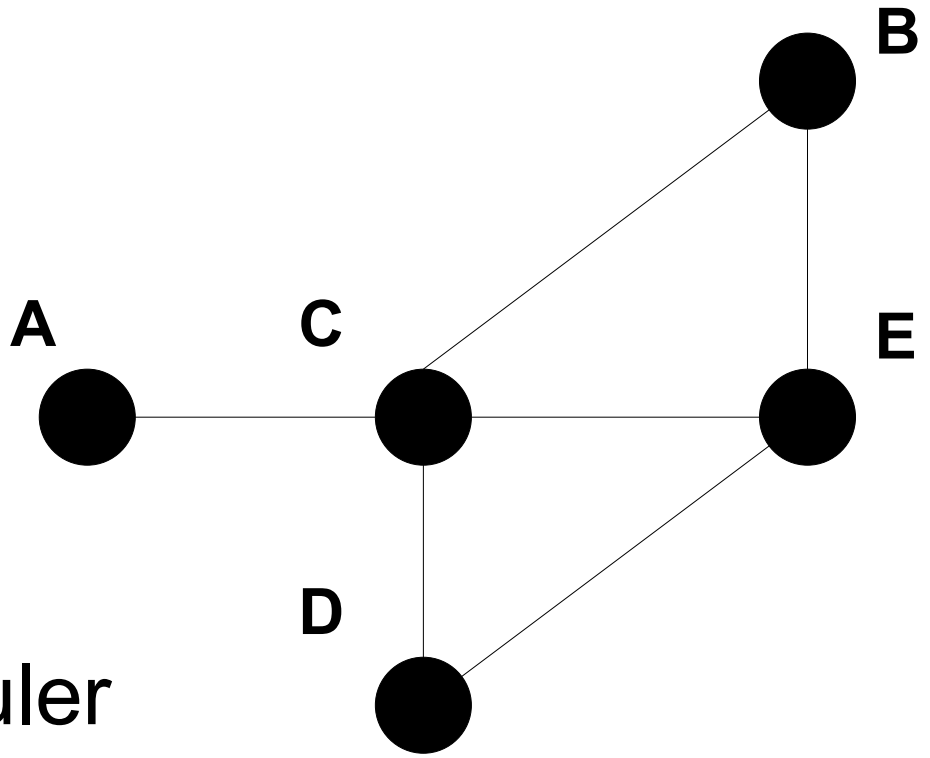
A graph can be **traversed** if there is an Euler path.

There is an Euler path if 0 or 2 verticies are odd.



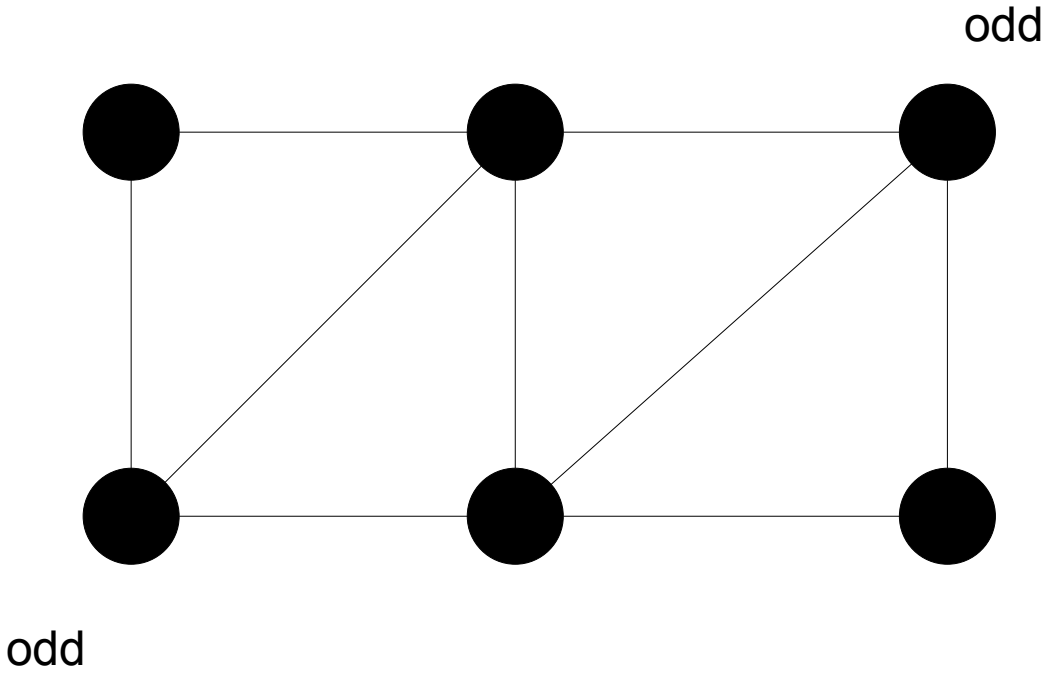
Pop quiz!!!

1) Find an odd vertex.

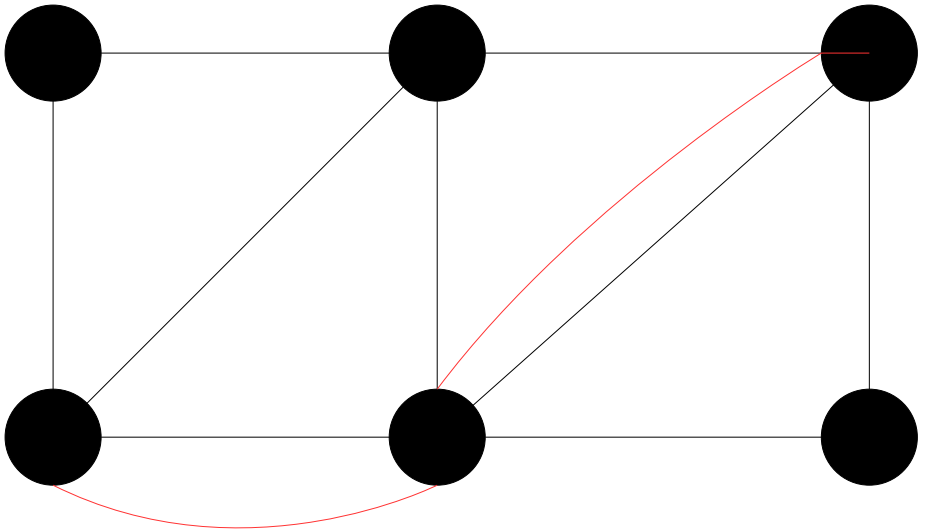


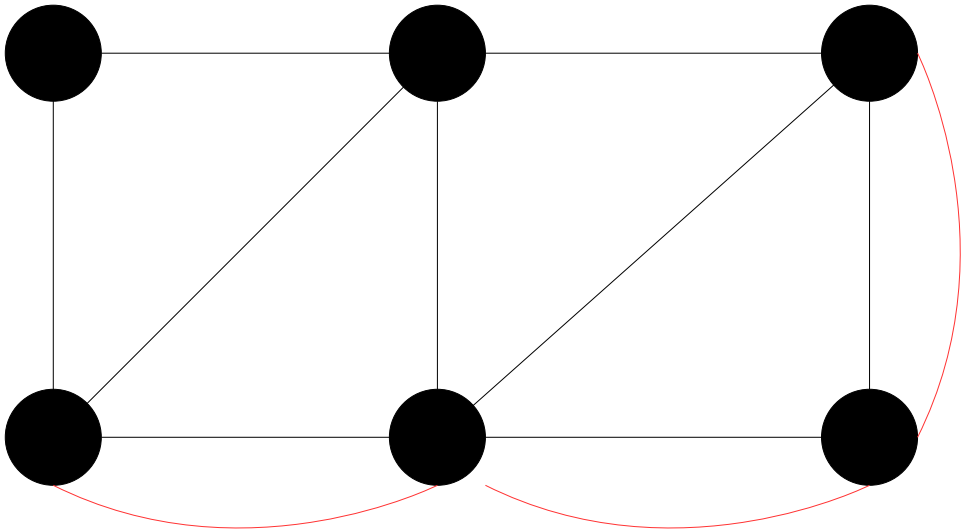
2) Is there an Euler path?

Eulerizing is the process of duplicating edges until all verticies are even.



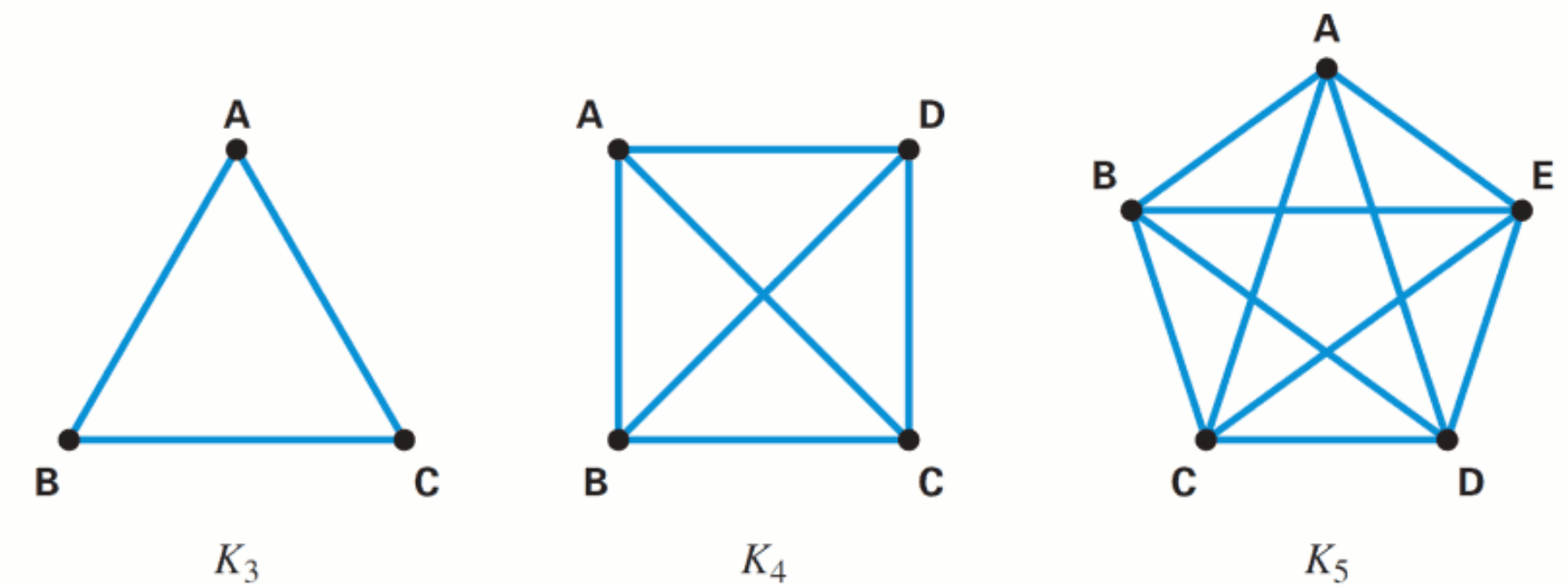
Two solutions. Which is more optimal?





A **complete graph** has ...

A **complete graph** has every vertex connected to every other vertex.



A Hamiltonian path goes through each _____ once.

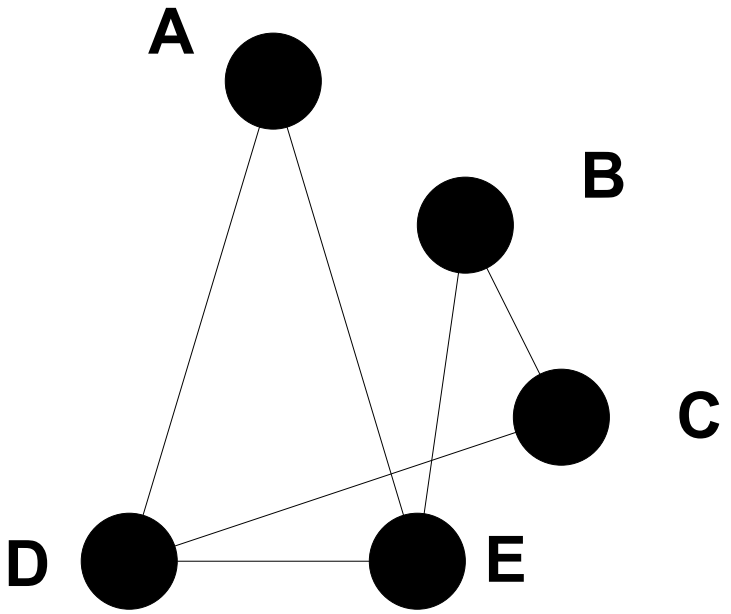
A Hamiltonian path goes through each vertex once.

So a Hamiltonian path might miss edges.

An Euler path goes through each edge once.

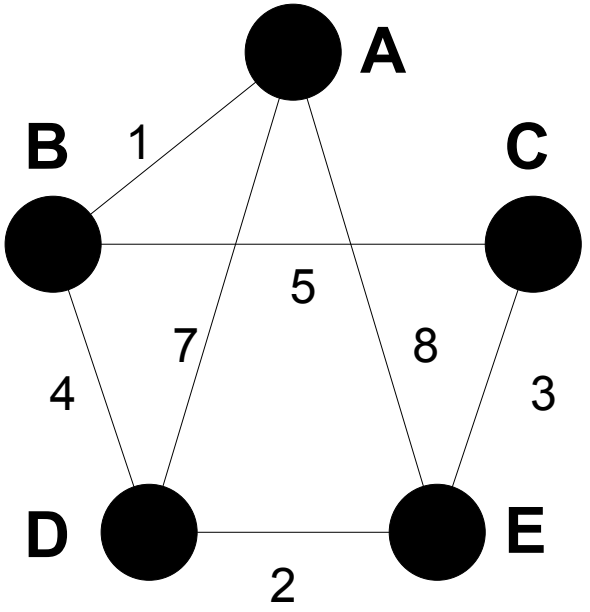
It might use the same vertex multiple times.

A **circuit** (Euler or Hamiltonian) is a path that starts and ends at the same vertex.



The **weight** of an edge is a number assigned to the edge. (Think distance between cities.)

A graph is a **weighted graph** if all of its edges have weights.

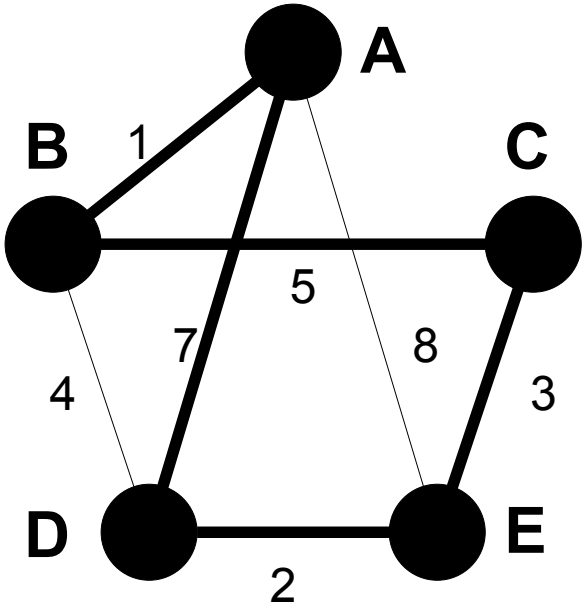


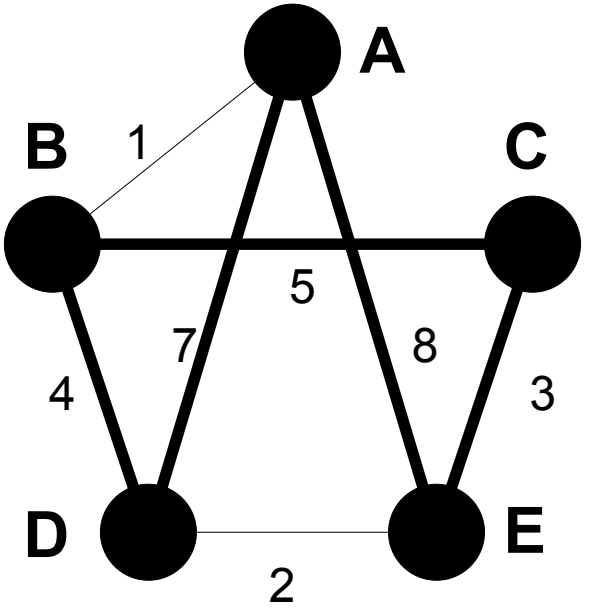
What is the weight of

a) A,B
b) C,B
c) C,D - no edge, no weight

Traveling Salesman Problem

- visit every city (vertex) with least distance (weight)
- so least weight Hamiltonian path.





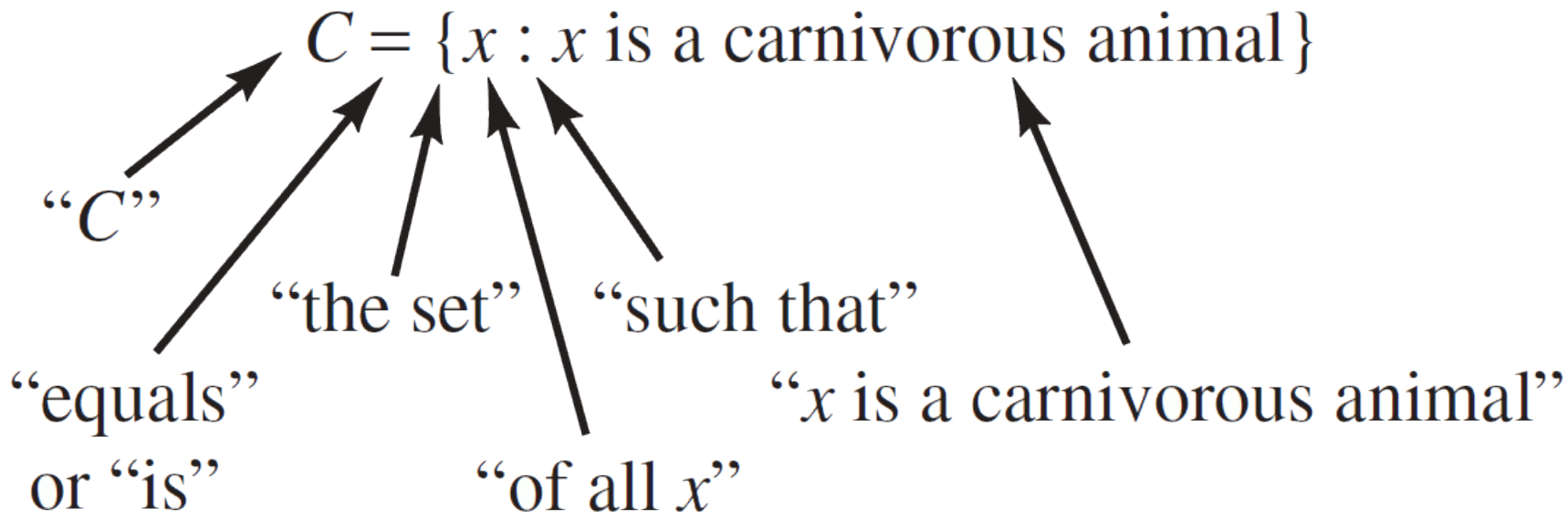
Pop quiz!!!

Hamiltonian Paths visit each ___ exactly once.

1) Circuit
2) Edge
3) Complete
4) Vertex
5) None of the above

Representing Sets

- Set-builder notation:



{ 1, 2, 3, 4, … 10 }

{ x | x is positive and even }

{ University Hall, Snyder Memorial, Gillham Hall, Field House, Rocket Hall, Palmer, ... }

A set with no entries is known as the empty set. It can also be written as

$\varnothing$

The empty set is a subset of every set.

It is not an element of every set, but here is an example:

A = { Bob, 12345, $\varnothing$, pumpkins }

n(A) = the number of elements in set A

A = { Bob, 12345, ∅, pumpkins }
n(A) =

B = { a, b, c, … x, y, z }
n(B) =

C = { x | x is a day of the week }
n(C) =

$\in$ means "is an element of"

$\notin$ means "is *not* an element of"

Pop Quiz!!!!

A = { 1, 2, 3, 4, 5}

B = { {1}, {2}, {3}, {4}, {5} }

1) 5 is an element of

2) {5} is an element of

3) {5} is a subset of

Venn Diagrams and Proper Subsets

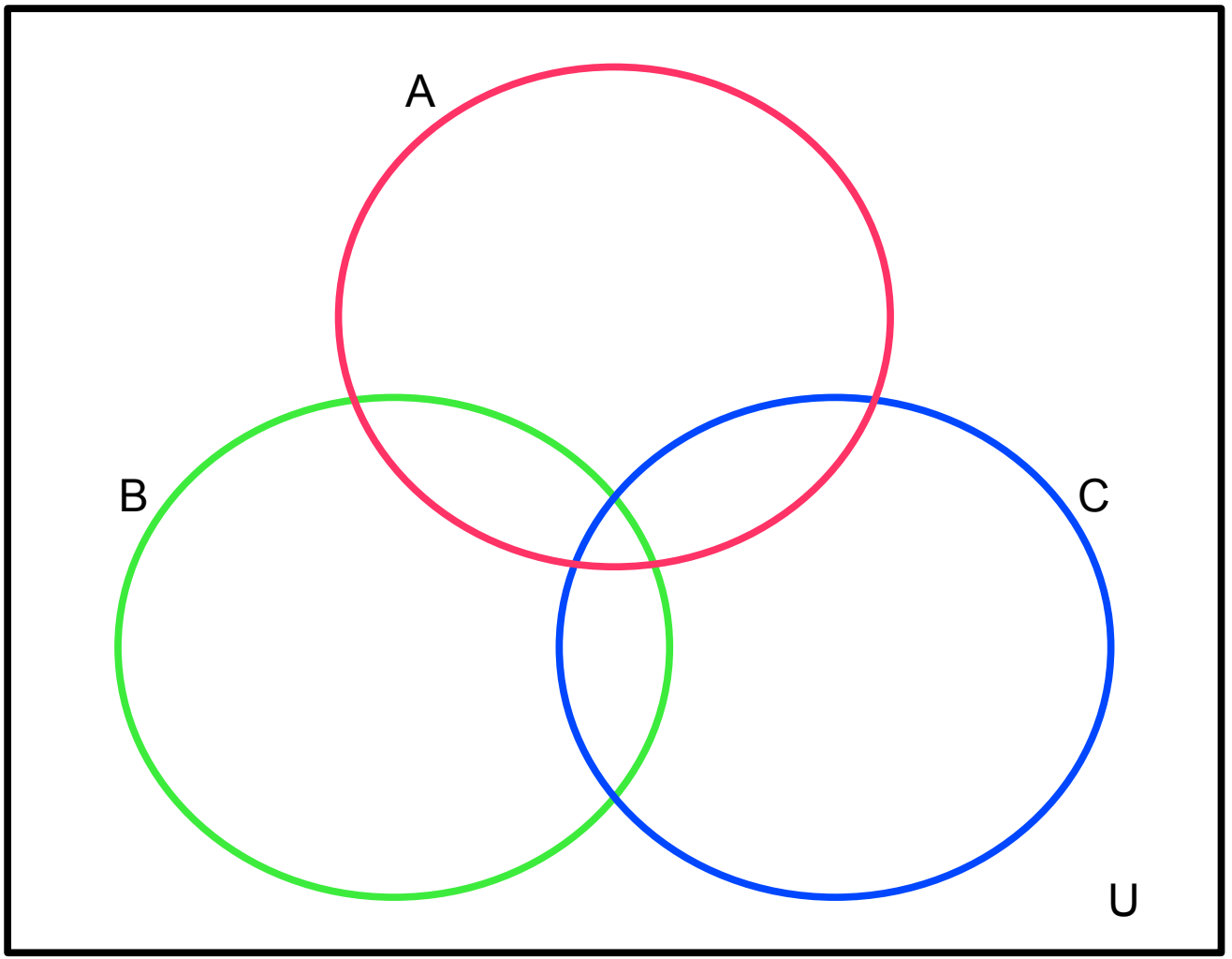
THE NUMBER OF SUBSETS OF A SET A set that has k elements has 2^k subsets.

- How many subsets exist for the given set?

$$A = \{\text{Bill, Gill, Jill, Will}\}$$

$$2^k = 2^4 \quad 16$$

Venn Diagram.



Union of Sets

DEFINITION The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of either A or B (or both). Using set-builder notation,

$$A \cup B = \{x : x \text{ is a member of } A \text{ or } x \text{ is a member of } B\}.$$

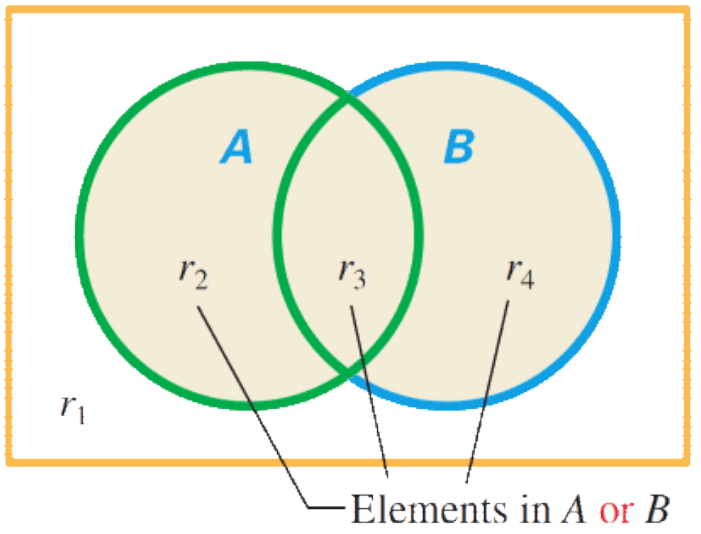
The union of more than two sets is the set of all elements belonging to at least one of the sets.

- Example: Find the union of the two sets.

$$A = \{1, 3, 5, 6, 8\}$$

$$B = \{2, 3, 6, 7, 8\}$$

$$A \cup B = \{1, 2, 3, 5, 6, 7, 8\}$$



Intersection of Sets

DEFINITIONS The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both A and B. Using set-builder notation,

$$A \cap B = \{x : x \text{ is a member of } A \text{ and } x \text{ is a member of } B\}.$$

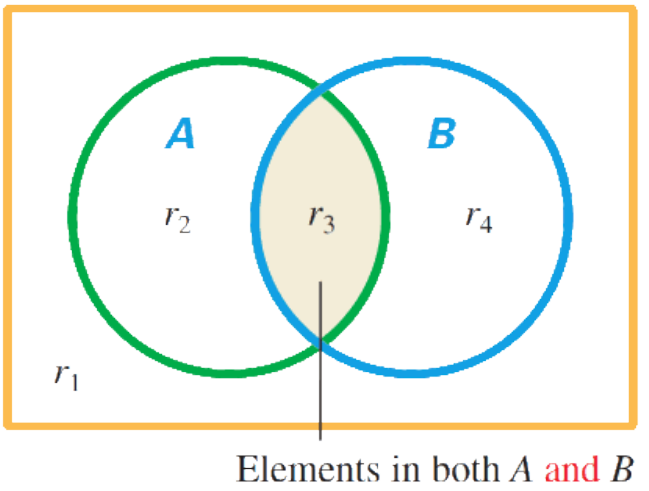
The intersection of more than two sets is the set of elements that belong to each of the sets. If $A \cap B = \varnothing$, then we say that A and B are **disjoint.**

- Example: Find the intersection of the two sets.

$$A = \{1, 3, 5, 6, 8\}$$

$$B = \{2, 3, 6, 7, 8\}$$

$$A \cap B = \{3, 6, 8\}$$



Set Complement

DEFINITION If A is a subset of the universal set U, the **complement** of A is the set of elements of U that are *not* elements of A. This set is denoted by A'. Using set-builder notation,

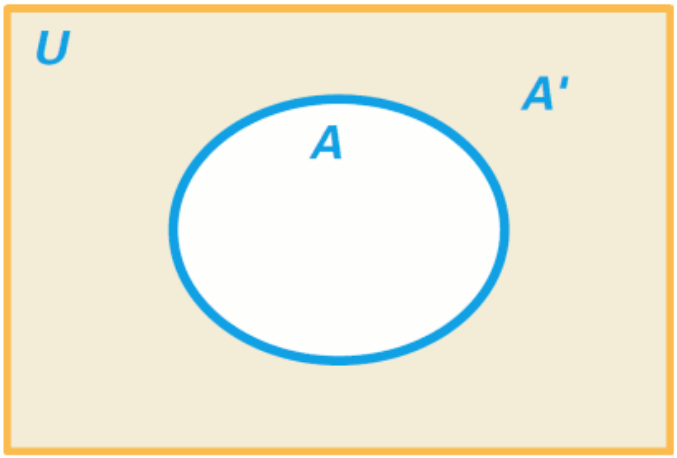
$$A' = \{x : x \in U \text{ but } x \notin A\}.$$

- Example: Given U, find the complement of A.

$$U = \{1, 2, 3, \ldots, 10\}$$

$$A = \{1, 3, 5, 7, 9\}$$

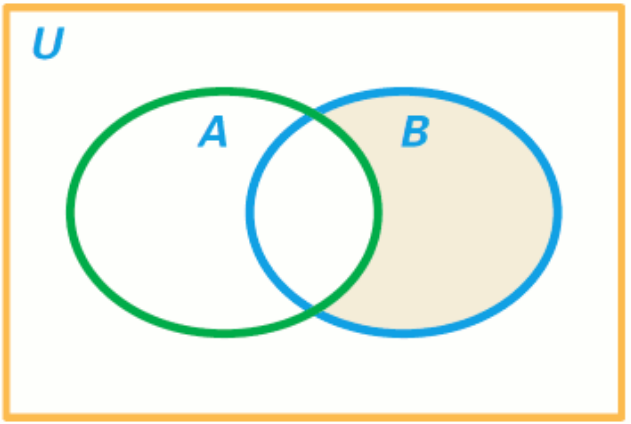
$$A' = \{2, 4, 6, 8, 10\}$$



Set Difference

DEFINITION The **difference** of sets B and A is the set of elements that are in B but not in A. This set is denoted by $B - A$. Using set-builder notation,

$$B - A = \{x : x \text{ is a member of } B \text{ and } x \text{ is not a member of } A\}.$$



- Example: Find the difference.

$$\{3, 6, 9, 12\} - \{x : x \text{ is an odd integer}\}$$

$\{3, 6, 9, 12\}$ and remove all the odd integers to get $\{6, 12\}$

Remove elements that are odd.

Order of Operations

( )	parenthesis always done first.
'	set complement next.
U, ∩, –	union, intersection, difference last

Methods of solving

- Venn Diagrams

- Shorthand

  B' =

  A U B' =

- Longhand

  { } U { }' =

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }
C = { 3, 4, 5 }

Pop Quiz!!!
How many elements in $A \cap B$?

Find $(A \cup B) - C'$

a | b means “a divides b”

5 | 30

7 | 7

If a | b then b = a c where c is some other number. This is a factor of b.

20 | 1000 and 1000 / 20 = 50, so

1000 = 20 x 50

A number who's factors are only 1 and itself is a **prime** number.

2, 3, 5, 7, 11, 13, 23, 29, 31, 37, etc

You can use a Sieve of Eratosthenes to find them.

A **factor tree** splits a number into 2 factors at each step.

Example: 420

The prime factorization is the collection of all the primes.

GCD = greatest common divisor
= use lowest power of factors

$2^8\ 3^2\ 5\ 11$
$2^2\ 3\ 5\ 7$

LCM = least common multiple
= use largest power of factors

$2^8\ 3^2\ 5\ 11$
$2^2\ 3\ 5\ 7$

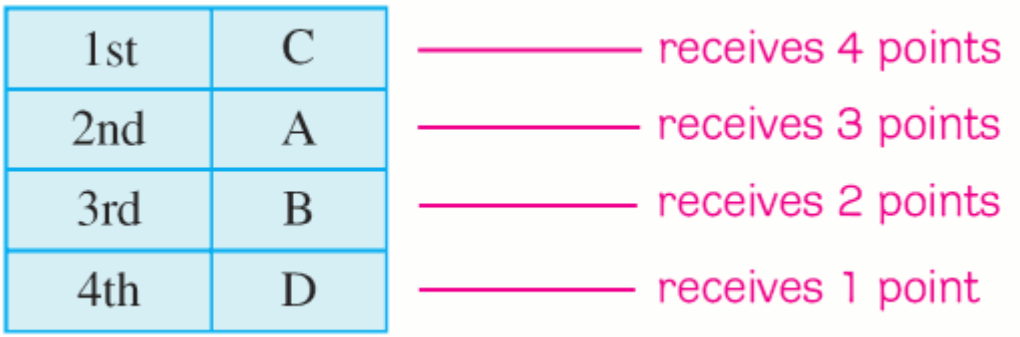
There are different voting methods when you have more than 2 candidates.

1) Plurality
2) Borda Count
3) Plurality with Elimination
4) Pairwise Comparison

Plurality

Person with most votes wins.

Borda Count



Add up points for each candidate. The one with the most points wins.
On a ballot, a last place vote gets 1 point, second to last place vote gets 2 points, etc.

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Plurality with Elimination

Remove candidate with least 1st place votes.
Retally ballots.
Repeat until 1 person remains.
This person is the winner.

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Pairwise Comparison

Compare every pair of candidates.
If one wins, they get 1 point, the other 0.
If they tie, they both get ½ points.

The candidate with the most points wins.

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Fairness Conditions and Criteria for analyzing defects.

DEFINITION **The Majority Criterion**
If a majority of the voters rank a candidate as their first choice, then that candidate should win the election.

DEFINITION **Condorcet's Criterion**
If candidate X can defeat each of the other candidates in a head-to-head vote, then X is the winner of the election.

DEFINITION **Independence-of-Irrelevant-Alternatives Criterion**
If candidate X wins an election, some nonwinners are removed from the ballot, and a recount is done, then X still wins the election.

DEFINITION **The Monotonicity Criterion**
If X wins an election and in a reelection all voters who change their votes only change their votes to favor X, then X also wins the reelection.

No current voting method satisfies all of these well-meaning conditions and criteria

Pop quiz!!!!!

Which of these is not a voting method?

1) Borda Count
2) Plurality with Elmination
3) Plurality
4) Majority Criterion
5) Pairwise Comparison

DEFINITIONS A **weighted voting system** with n voters is described by a set of numbers that are listed in the following format:

[quota: weight of voter 1, weight of voter 2, . . . , weight of voter n]

The **quota** is the number of votes necessary in this system to get a resolution passed. The numbers that follow, called **weights**, are the amount of votes controlled by voter 1, voter 2, etc.

Example:

[10: 2, 2, 2, 4, 4]

Any voters that vote the same way is called a **coalition**.

A **winning coalition** is a coalition that can always pass an issue / meets the quota.

Example: Find a winning coalition of voters.

[10: 2, 2, 4, 4, 4]

DEFINITION A voter in a winning coalition is called **critical** if it is the case that if he or she were to leave the coalition, then the coalition would no longer be winning.

Who is critical to get 8 votes?

{A}	5	
{B}	3	
{C}	4	
{A,B}	8	winning
{A,C}	9	winning
{B,C}	7	
{A,B,C}	12	winning

Compute the Banzhaf Power Index for A, B, C.

			critical
{A}	5		
{B}	3		
{C}	4		
{A,B}	8	winning	A, B
{A,C}	9	winning	A, C
{B,C}	7		
{A,B,C}	12	winning	A

Compute the Banzhaf Power Index for A, B, C.

			critical	
{A}	5			
{B}	3			
{C}	4			
{A,B}	8	winning	A, B	(Note: total
{A,C}	9	winning	A, C	critical voters
{B,C}	7			is 5 = 3+1+1)
{A,B,C}	12	winning	A	

A critical 3 times, B critical 1 time, C critical 1 time

Banzhaf Power Index

A : 3/(3+1+1) B : 1/(3+1+1) C : 1/(3+1+1)